The tipping point: A mathematical model for the profit-driven abandonment of restaurant tipping

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The custom of voluntarily tipping for services rendered has gone in and out of fashion in America since its introduction in the 19th century. Restaurant owners who ban tipping in their establishments often claim that social justice drives their decisions, but we show that rational profit-maximization may also justify the decisions. Here, we propose a conceptual model of restaurant competition for staff and customers, and we show that there exists a critical conventional tip rate at which restaurant owners should eliminate tipping to maximize profits. Because the conventional tip rate has been increasing steadily for the last several decades, our model suggests that restaurant owners may abandon tipping en masse when that critical tip rate is reached. Published by AIP Publishing.

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I. INTRODUCTION

Tipping for restaurant services has gone in and out of fashion in America since its introduction from Europe in the 19th century.1 Tipping has always been a controversial social convention, for both scholars and the public. The practice has been consistently tied to the worst of human nature: racism, sexism, and classism.2–5 At many points in time, tipping has been considered downright anti-democratic.6 Yet, the practice persists because the vast majority of Americans prefer to choose how much gratuity they leave after a meal.7,8

Economists have traditionally struggled to explain the practice of tipping in terms of rational costs and benefits because a rational economic agent would not incur a monetary cost that provides no present or future benefit.9 Sociologists and psychologists have appealed to negative feelings of guilt, embarrassment, or anxiety to explain why people conform to the social convention of tipping.10–12

while others have appealed to altruistic feelings of generosity and empathy.13–15

While much scholarly attention has been paid to the consumers who tip and employees who receive tips (e.g., Refs. 16–19), relatively little has been paid to restaurant owners who employ tipped staff. Theoretical models of tipping often see restaurant owners as inefficient judges of service quality.20 The natural conclusion to this rationale is that restaurant owners should allow tips in their restaurants in order to efficiently evaluate the quality of their wait staff.21 This conclusion breaks down when we consider the real-world factors that influence restaurant owners’ decisions, such as remaining profitable, while also adhering to minimum wage restrictions, retaining talented staff, and meeting customer expectations for food and service quality.22 By accounting for these factors and appealing only to rational profit-maximization motivations, we show that restaurant owners should play a more active role in determining tip rates in their restaurants.

As the conventional tip rate gradually increases in the U.S. (see Fig. 1), waiters’ take home pay steadily increases, while back-of-house employees’ pay remains stagnant.23 Despite the low federal minimum wage for tipped workers ($2.13 as of 2017, Ref. 24), waiters consistently earn more than cooks.25 As the wage disparity increases, talented cooks may defect to restaurants where profits are shared more equitably among staff, and talented waiters may defect to restaurants with higher tips. A rational restaurant owner interested only in maximizing the profit might take control of the tip rate in his/her restaurant in order to retain the most talented front-of-house (tipped) and back-of-house (untipped) staff. We show in a conceptual model of two competing restaurants that a critical tipping rate exists at which a rational restaurant owner will abandon tipping to maximize profits.
two restaurants. Assuming for simplicity that the transition probabilities are linear in the relative base cook pay, the change in the number of cooks $\hat{C}$ at our restaurant is

$$\tau_C \frac{d\hat{C}}{dt} = \left( N_C - \hat{C} \right) \frac{b_{C1}}{b_{C1} + b_{C2}} - \hat{C} \frac{b_{C2}}{b_{C1} + b_{C2}},$$

(2)

where $\tau_C$ sets the transition time scale for cooks and $b_{C1}$ and $b_{C2}$ are the hourly base cook pay at our restaurant and the other restaurant, respectively. For example, if both restaurants offer the same base pay ($b_{C1} = b_{C2}$), then eventually half the cooks will be at our restaurant, and the other half will go to the competitor. Tilde notation will be removed later when the model is normalized.

2. Dynamics of waiters

Like the cooks, the waiters in our system are primarily concerned with pay. Because waiters receive both hourly wages and gratuity, the transition rate between restaurants depends on the relative hourly take home (total) pay at the two restaurants. The hourly gratuity at our restaurant is

$$g_1(D, W) = \frac{m_1 DT_1}{W},$$

(3)

where $m_1$ is the hourly menu price, $D$ is the number of diners, $T_1$ is the tip rate, and $W$ is the number of waiters that must split the total tips. The gratuity $g_2$ at the competing restaurant is similarly defined.

The hourly take home pay at our restaurant is then $b_{W1} + g_1$, where $b_{W1}$ is the hourly base waiter pay at our restaurant. The change in the number of waiters $\tilde{W}$ at our restaurant is then

$$\tau_W \frac{d\tilde{W}}{dt} = \left( N_W - \tilde{W} \right) \frac{b_{W1} + g_1}{b_{W1} + g_1 + b_{W2} + g_2},$$

(4)

where $\tau_W$ sets the transition time scale for waiters and $N_W$ is the number of waiters in the system.

3. Dynamics of diners

Assuming that all diners intend to eat at a restaurant, they must choose between our restaurant and our competitor. Many factors influence a person’s decision to eat at a particular restaurant, but we will focus on food and service quality versus menu cost. There are also many ways to measure food and service quality, but we will use the number of cooks and waiters who choose to work at our restaurant as a basic proxy. For instance, if our restaurant attracts more waiters, then diners will receive more personal attention and
perceived service quality will increase. Suppose for simplicity that the quality \( q_1 \) of the meal and service at our restaurant is a linear combination of the number of cooks \( \hat{C} \) and waiters \( \hat{W} \) working at our restaurant
\[
q_1(\hat{W}, \hat{C}) = \alpha_W \hat{W} + \alpha_C \hat{C},
\]
where \( \alpha_W \) and \( \alpha_C \) are the weights placed on the service and food, respectively, when evaluating our restaurant. The quality \( q_2 \) of the competing restaurant is defined similarly. Note that this proxy for restaurant quality will only be an acceptable approximation if both restaurants are not grossly over- or under-staffed. We explore two alternative formulations for restaurant quality in the supplementary material, and we find similar qualitative results.

We define the value \( v_1 \) of our restaurant as the quality \( q_1 \) over the menu cost (including tips)
\[
v_1(\hat{W}, \hat{C}) = \frac{\alpha_W \hat{W} + \alpha_C \hat{C}}{m_1(1 + T_1)} ,
\]
where \( m_1 \) is the hourly menu cost and \( T_1 \) is the tip rate at our restaurant. The value \( v_2 \) of the other restaurant is defined similarly.

A rational diner chooses a restaurant based on the perceived relative value of each restaurant. The change in the number of diners \( D \) at our restaurant is then
\[
\tau_D \frac{dD}{dt} = (N_D - D) \frac{v_1}{v_1 + v_2} \frac{\hat{D}}{\hat{v}_1 + \hat{v}_2} - \frac{\hat{D}}{\hat{v}_1 + \hat{v}_2} \frac{v_2}{v_1 + v_2} ,
\]
where \( \tau_D \) sets the transition time scale for diners and \( N_D \) is the number of diners in the system.

### 4. Profitability

Given the flow of employees and customers to and from our restaurant, a rational restaurant owner will maximize hourly profit
\[
\hat{P}(D, W, C) = m_1 \hat{D} - b_{W1} \hat{W} - b_{C1} \hat{C} .
\]
We ignore fixed costs because we are only concerned with maximizing profitability, not absolute profits.

### B. Normalized model

We now normalize and nondimensionalize the system (2)–(7) to reduce the number of parameters. We make the following substitutions:
\[
D = \frac{\hat{D}}{N_D}, \quad W = \frac{\hat{W}}{N_W}, \quad C = \frac{\hat{C}}{N_C},
\]
\[
r = \frac{\alpha_C}{\alpha_W}, \quad r_{DW} = \frac{N_D}{N_W}, \quad r_{CW} = \frac{N_C}{N_W},
\]
so that \( D, W, \) and \( C \) are the fraction of diners, waiters, and cooks at our restaurant, \( r \) is the ratio of food to service importance for customers, and \( r_{DW} \) and \( r_{CW} \) are the ratios of diners and cooks to waiters, respectively. We also rescale time such that \( \tau_C = \tau_W = \tau_D = 1 \). Naturally, the transition rates may vary for diners, waiters, and cooks; customers may switch dining locations more rapidly than employees switch jobs. However, we are only interested in equilibrium states, and so, we ignore this detail.

Then, the fraction of diners at our restaurant follows the dynamics
\[
\frac{dD}{dt} = (1 - D) \frac{v_1}{v_1 + v_2} - D \frac{v_2}{v_1 + v_2} ,
\]
\[
v_1(W, C) = \frac{W + r_{CW} \quad C}{m_1(1 + T_1)} ,
\]
\[
v_2(W, C) = \frac{(1 - W) \quad + \quad r_{CW} \quad (1 - C)}{m_2(1 + T_2)} .
\]
The fraction of waiters at our restaurant follows the dynamics
\[
\frac{dW}{dt} = (1 - W) \frac{b_{W1} + g_1}{b_{W1} + g_1 + b_{W2} + g_2} - W \frac{b_{W2} + g_2}{b_{W1} + g_1 + b_{W2} + g_2} ,
\]
\[
g_1(D, W) = \frac{m_1 r_{DW} D T_1}{W}, \quad g_2(D, W) = \frac{m_2 r_{DW} (1 - D) T_2}{1 - W} .
\]
Finally, the fraction of cooks at our restaurant follows the dynamics
\[
\frac{dC}{dt} = (1 - C) \frac{b_{C1}}{b_{C1} + b_{C2}} - C \frac{b_{C2}}{b_{C1} + b_{C2}} .
\]
All variables and parameters are described in Table I.

With this change of variables, hourly profit \( \hat{P} \) becomes the hourly profit per waiter in the system
\[
P(D, W, C) = m_1 r_{DW} D - b_{W1} W - b_{C1} r_{CW} C .
\]

### III. RESULTS

#### A. Numerical exploration

Numerical integration suggests that one stable steady state solution exists for each set of parameters regardless of the initial condition, as long as the initial condition is physically meaningful. Because cooks only switch restaurants in response to base pay (constant parameter), the distribution of cooks equilibrates first. Diners and waiters respond to every- one else in the system, and so, the distribution of diners and waiters equilibrates later.

For otherwise identical restaurants, small changes in restaurant policy (like staff pay, menu prices, and tip rates) will have an effect on the entire restaurant ecosystem. Lowering the tip rate at our restaurant will cause waiters to leave our restaurant because they get paid less, but diners will prefer our restaurant because they pay less [see Fig. 2(a)].
TABLE I. Description of model variables and parameters for our restaurant, Restaurant 1. The competing restaurant (Restaurant 2) has similarly defined parameter values subscripted with 2. We present a range of plausible values for each parameter and a baseline value for midscale and upscale restaurants like those reviewed by Zagat.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Meaning</th>
<th>Units</th>
<th>Range</th>
<th>Baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>Fraction of diners at our restaurant</td>
<td>...</td>
<td>[0, 1]</td>
<td>...</td>
</tr>
<tr>
<td>W</td>
<td>Fraction of waiters at our restaurant</td>
<td>...</td>
<td>[0, 1]</td>
<td>...</td>
</tr>
<tr>
<td>C</td>
<td>Fraction of cooks at our restaurant</td>
<td>...</td>
<td>[0, 1]</td>
<td>...</td>
</tr>
<tr>
<td>t</td>
<td>Time (dimensionless)</td>
<td>...</td>
<td>[0, ∞)</td>
<td>...</td>
</tr>
<tr>
<td>r</td>
<td>Relative importance of food quality versus service quality</td>
<td>...</td>
<td>[4, 20]</td>
<td>12²</td>
</tr>
<tr>
<td>rCW</td>
<td>Ratio of total cooks to waiters in the system</td>
<td>...</td>
<td>[0.5, 2]</td>
<td>1</td>
</tr>
<tr>
<td>rDW</td>
<td>Ratio of total diners to waiters in the system</td>
<td>...</td>
<td>[1, 20]</td>
<td>12</td>
</tr>
<tr>
<td>m1</td>
<td>Average menu cost per hour at our restaurant</td>
<td>$/h</td>
<td>[5, 20]</td>
<td>10</td>
</tr>
<tr>
<td>bW1</td>
<td>Waiters’ base pay per hour at our restaurant</td>
<td>$/h</td>
<td>[2.13³, 25]</td>
<td>5.00⁵</td>
</tr>
<tr>
<td>bc1</td>
<td>Cooks’ base pay per hour at our restaurant</td>
<td>$/h</td>
<td>[7.25⁶, 25]</td>
<td>10.40⁶</td>
</tr>
<tr>
<td>T1</td>
<td>Average tip rate at our restaurant, determined by either social convention or mandated by restaurant owner</td>
<td>...</td>
<td>[0.01, 0.5]</td>
<td>0.19⁴</td>
</tr>
<tr>
<td>v1</td>
<td>Meal value perceived by customers at our restaurant</td>
<td>1/$</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>g1</td>
<td>Gratuity per hour at our restaurant</td>
<td>$/h</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

²Crude estimate based on customer surveys.²⁸
³Federal minimum wage as of 2017.²⁴
⁴Average waiter and cook pay as of 2015.³¹
⁵Average self-reported tip rate as of 2016,²⁶ and other baseline values are guesses based on author experience.

If the menu price at our restaurant is lower than the competitor, then diners will flock to our restaurant because they pay less, and waiters will temporarily leave our restaurant because lower menu prices lead to lower tips. However, after our restaurant has a large share of diners, waiters return because the density of diners balances the lower menu prices [see Fig. 2(b)].

![Fig. 2](image_url)

**FIG. 2.** Numerical simulation of systems (11)–(15) with two nearly identical competing restaurants. (a) If the tip rate at our restaurant is lower than the competitor, then waiters will leave, but diners will prefer our restaurant ($T_2 = 0.25$). (b) If the menu price at our restaurant is lower than the competitor, then diners will flock to our restaurant, and waiters will temporarily leave. However, after our restaurant has a large share of diners, waiters will return ($m_2 = 15$). (c) If we pay our cooks less, then cooks followed by diners followed by waiters will leave ($b_{c2} = 12$). (d) If we pay our cooks more but pay our waiters less to compensate, cooks will flock to our restaurant followed by diners; waiters will temporarily leave because they are paid lower wages, but eventually they will come back as diners flood our restaurant ($b_{c2} = 10, b_{s1} = 15$). Unless otherwise noted, $m_1 = m_2 = 10, T_1 = T_2 = 0.2, b_{c1} = b_{c2} = 5, b_{s1} = b_{s2} = 10, r = 12$. If we pay our cooks less than our competitor, then cooks will leave our restaurant because they get paid less; as food quality decreases, diners will leave our restaurant, and then, waiters will leave our restaurant as their hourly tips decrease [see Fig. 2(c)].

As a final example, if we pay our cooks more but pay our waiters less to compensate, cooks will flock to our restaurant followed by diners; waiters will temporarily leave because they are paid lower wages, but eventually they will come back as diners flood our restaurant [see Fig. 2(d)].

**B. Equilibrium stability analysis**

Fixed point analysis shows that four steady states exist. Only one fixed point is meaningful (i.e., $D^*, W^*, C^* \in [0, 1]$). The steady state for cooks is $C^* = b_{c1}/(b_{c1} + b_{c2})$. The steady states for waiters and diners have closed forms but are too long to include. For all reasonable parameter values (listed in Table I), the eigenvalues of the Jacobian evaluated at the fixed point are real and negative. This implies that the equilibrium is a stable sink (see Fig. S5).

**C. Equilibrium sensitivity analysis**

Global sensitivity and uncertainty analysis using Latin Hypercube Sampling (LHS) of parameter space and Partial Rank Correlation Coefficients (PRCCs)³² reveals that equilibrium distributions of diners and waiters depend significantly ($p < 0.001$) on tip rates and cook pay. Equilibrium distributions of waiters also depend significantly on waiter pay. Note that the parameters that significantly influence these distributions describe the differences between restaurants and do not describe the system as a whole (see Fig. S6).
IV. DISCUSSION

A. Tip abandonment threshold

Suppose that our restaurant is attempting to maximize the hourly profit (16) at equilibrium. We assume that our restaurant is competing with a typical American restaurant that is not making dynamic changes to staff pay, menu prices, or tipping policies. Given the choices the other restaurant has made, our restaurant can choose base pay for cooks and waiters (within legal limits) and a gratuity policy. Both restaurants maintain identical menu prices to ensure that the restaurants are true competitors; fine dining establishments do not typically compete with casual restaurants.

If the competing restaurant allows the conventional tipping rate, then there exists a critical tip rate threshold $T_c$ at which a rational restaurant owner would forbid tipping to maximize profits. Figure 3(a) shows the conventional tip rate at which a hypothetical restaurant should switch from allowing the conventional tip to abandoning tipping in their establishment. Although the critical tip rate depends on the entire restaurant ecosystem, numerical exploration indicates that the trade-off between meal cost and restaurant quality, as perceived by diners, is the primary driver of the critical tip rate [see Figs. 3(b)–3(f)]. Assuming that the conventional tip rate continues to increase in the U.S., we predict that restaurants will eventually forbid tipping when it becomes more profitable to do so.

Global sensitivity and uncertainty analysis shows that this critical tipping threshold depends significantly on the menu price shared by both restaurants, the ratio of customers to waiters and cooks to waiters, and the ratio of food quality to service quality in the eyes of the customer (see Fig. 4).

![Graphs showing various tipping scenarios](https://example.com/figure3.png)

**Fig. 3.** Example of the critical tip rate threshold. (a) For conventional tip rates below some critical threshold $T_c$, a rational restaurant owner would allow diners to leave gratuity to maximize profitability (black curve). Beyond that critical threshold, a rational restaurant owner would disallow tipping in their restaurant (red dashed curve). Both curves assume that the restaurant owner selects staff pay (within legal limits) to maximize profits. For this hypothetical restaurant ecosystem, (b) both optimal cook pay and (c) total waiter pay (optimal wage plus tips) drop if we eliminate tipping. Although staff leave our restaurant in response to a no-tip policy, (d) the drop in perceived quality balances (e) the effective menu price (menu cost plus tips) near the critical tip rate. (f) The trade-off between quality and price, as perceived by diners, drives the critical tip rate. For this example, $m_1 = m_2 = 10, r = 4, b_{W1} = 10, b_{C2} = 25, r_{DW} = 10, r_{CW} = 1$, the minimum wage for tipped workers is 2.13, and the minimum wage for untipped workers is 7.25.
Note that the parameters that significantly influence the critical tip rate \( T_c \) describe the type or “class” of restaurant system we are considering. For instance, fine dining restaurants maintain a low diner to waiter ratio \( r_{DW} \) and high menu prices \( m \). It is also likely that diners at fine dining establishments place more value on service than at casual restaurants, decreasing \( r \).

Local sensitivity analysis about ‘typical’ American restaurant parameters suggests that the increased menu price, increased service importance, increased diner-to-waiter ratio, and increased waiter-to-cook ratio all increase the critical tipping rate (see Fig. 5). Because no type or class of restaurant increases all these parameters, we cannot say with certainty that a certain type of restaurant should abandon tipping before another. However, the three strongest correlated parameters \( (r, r_{CW}, m) \) support the prediction that casual dining restaurants should be the first to abandon tipping, and fine dining establishments should be the last to abandon tipping.

This prediction is consistent with tipping practices in the most casual restaurants: customers in fast food restaurants and counter-service establishments are not typically expected to leave tips. Among restaurants that expect patrons to leave tips, the prediction is surprising because the most vocal advocates for eliminating tipping in America have been owners of upscale restaurants. However, fine dining restaurant owners cite social justice as the primary motive for eliminating tipping in their establishments. This claim is consistent with our prediction because many restaurateurs have been forced to reinstate tipping in their restaurants in order to remain profitable.

**B. Limitations**

As a conceptual model, systems (11)-(15) cannot offer quantitative predictions with confidence. One limitation of this model is the lack of competition among many restaurants or eating at home; this could be addressed by considering the “competing restaurant” as a pool of competition. Additionally, the model assumes that the benefit of more employees does not have diminishing returns. More realistically, restaurant food or service will only benefit from more employees up to a certain point; after the restaurant is fully staffed, more employees will be a waste of money and may even impede service.

Our model also ignores both the federal law that requires restaurant owners to supplement tipped worker wages if their hourly tips do not exceed the federal minimum wage and many state laws that impose larger minimum wages for tipped employees. We also do not provide a mechanism by which the conventional tip rate increases and merely assume that the increasing trend will continue; however, the increasing trend is supported by theoretical economic models. Finally and most importantly, this model assumes that humans behave rationally when spending or earning money, a false assumption common among economic models. Restaurant owners may choose to abandon or maintain tipping regardless of profit, citing economically irrational reasons or responding to irrational customer feelings.

In spite of these limitations, the qualitative prediction that a critical tipping threshold exists at which restaurant owners may abandon tipping is supported by previous trends.
both in America and internationally. Tipping has gone in and out of fashion around the world, and although customers normally drive the introduction (or reintroduction) of the trend, restaurant owners or governments typically end the practice.  

V. CONCLUSION

The conceptual model presented here takes a new direction towards understanding the complex service industry. The oscillating popularity of tipping has previously been attributed to social contagion and irrational responses to classism. Using a new approach to modeling the social convention of tipping, we show that rational decisions to maximize profits may drive the cycle of the tipping trend. We predict that there exists a critical tip rate threshold at which restaurant owners would be wise to eliminate tipping in their establishments. Furthermore, we expect that casual restaurants should be the first to abandon tipping, and fine dining restaurants should be the last.

The simplicity of the model does not allow for quantitative predictions, such as when tipping will go out of fashion in the U.S. or what the threshold tip rate will be. However, the model serves as a base for more sophisticated models and could direct economic data collection to better answer quantitative questions. This effort would be important not only to restaurant owners but also to economists, sociologists, policy makers and all people who play a role in or interact with the service industry.

SUPPLEMENTARY MATERIAL

See supplementary material for additional discussion and figures.

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All software (Matlab.m files) used to simulate the model is publicly available from the Northwestern ARCH repository (DOI:10.21985/N2DM26) at https://arch.library.northwestern.edu/concern/generic_works/gf06g273).

The authors declare no competing interests.